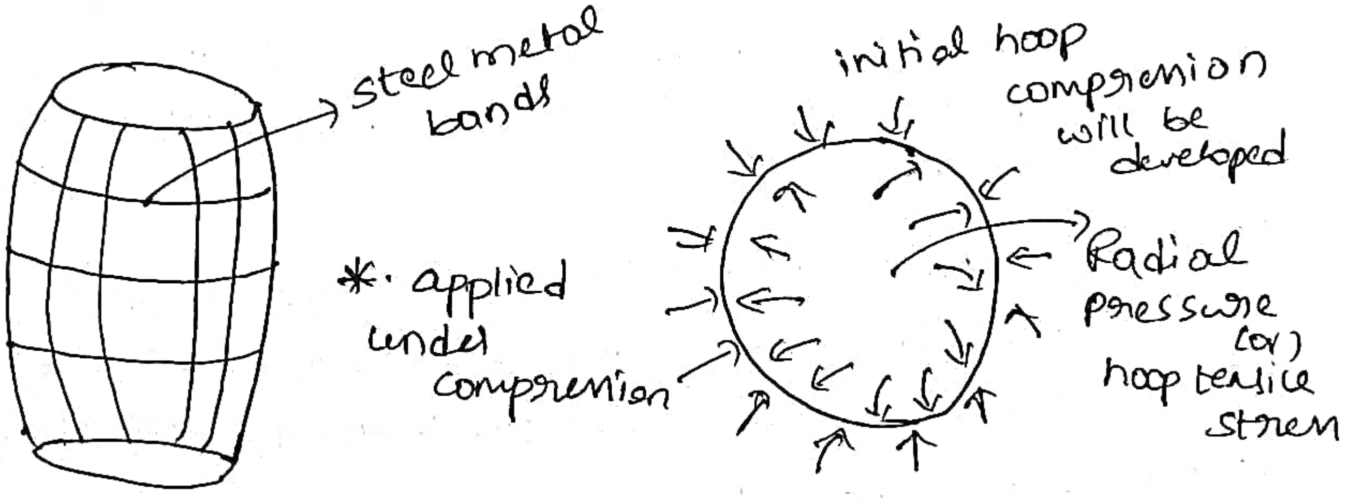
PRESTRESSED CONCRETE

1. Introduction

→ prestressed concrete is a R/F concrete that builts in the compressive stresses during construction to oppose those found when in use

NEED:-

→ To avoid cracks in structures like dome shaped
→ concrete @ tensile strength is only 8-14% of its compressive strength
→ To prevent cracks, compressive forces can be suitable application in the longitudnal direction either concentric (or) eccentric.
→ In cylindrical tanks, the hoop tensile stress can be effectively counted by circular prestressing.
→ It enhances the bending, shear & torsional capacity of flextural members.



Development (or) Brief history:

Cement was 1st introduced by Joseph Aspidin (england) - 1824 (portland)

→ 1857 - moines in france (R/F concrete) steel wires for (flower pots, pipes)

→ 1886 :- Jackson . pH (USA)

Introduced the concept of tightening steel tie rods in artificial stone & concrete arches.

→ 1888 : Doehring (Germany)

manufacturing of slabs & small beams with embedded tensioned steel

→ 1908 : Stainer (USA)

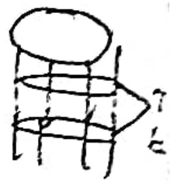
Recognised losses of due to shrinkage, creep & suggested to retightening of rods.

→ 1923 : Emperger (Austria)

Developed method of winding & pretensioning of high tensile steel wires around the pipes

→ 1924 :- Hewett (USA)

Hoop stress horizontal R/F around wall of concrete tanks use of turn buckles



→ 1925 :. Dill (USA)

used high strength unbonded steel rods. The rods are tensioned & anchored after hardening.

→ 1926 : Eugene freyssinet (France) (father of P.C.)

use "high tensile steel wire" with ultimate strength higher 1725 mpa & yield stress 1240 pa

→ 1939 : Development of conical wedges for end anchor for post tensioning

↳ wedge shaped

→ 1938 : Hayes (Germany)

developed "long line" pretensioning method & used some electrical forces

Eg: Railway sleepers

→ 1940 – magnel (Belgium)

Developed on anchoring system (or) post-tensioning using flat wedges

↳ flat wedges.

IN Building materials:-

The development of pre-stressed concrete can be study in the traditional building materials.

In ancient period stones and bricks were extensively used. These materials are strong in compre--ssion but weak in tension.

For tension bamboos are used in bridges subsequenty Iron and steel bars were used in to resist tension

wood and structural steel were effective in both tension & compression.

In r/f concrete at concrete & steel are combined such that concrete resist compression & tension. This is a passive combination.

In prestressed concrete high strength concrete and high tensioned steel are combined. such that the full section is effective in both tension and compression. This is an active combination.

Parameters:

1. wire: 2-7mm used
wire is a single unit made up of steel

2. strands: A no of steel wires grouped together by twisting.

3. Tendon:: A group of strands (or) wire are wound to form a prestressing tendon.

4. Bar: A Tendon can be made up of a single steel (—)
The diameter of bar is much longer than that of wire

5. cable: A group of tendons form a pre-stressing cable

6. Bonded tendon: where adequate bond b/n the pre-stressing tendon and concrete is called Bonded te. Pre-tensioned and grouted post-tensioned tendons are Bonded tendons.

7. unbonded tendons:- when there is no bond b/n the pre-stressing tendon and concrete it is called as unbonded tendon. when grout is not applied aft post tensioning the tendon is called unbonded tendons.

Advantages of prestressed concrete:-

1) section remains uncracked under service load:-
→ Suitable for use in pressure vessels, liquid reta structures like (water tank), dams, Gasoline pipes.
→ To avoid/ Reduction of steel corrosion as well as increase durability
→ Full section is utilised & less deformation is formed
→ Shear capacity must be high.

2) High span to depth ratio:-

AS per code IS 456: 2000 we have the ratio

R.C - 28:1 }
P.C = 45:1 } used mostly in Bridges.

→ longer spans possible with prestressing (Bridges, 64'b with long columns.

1-3

→ By taking typical "span/depth" ratio reduces the beam depth for very light members.

→ reduce the self weight
→ It should be economical
→ To increase the esthetic view
→ For some span less depth is provided compared to R.C members.

Suitable for precast construction:-

→ For rapid construction.
→ Better quality control.
→ Reduce maintainence cost.
→ Avoulability of standard shapes
→ Reduction of ~~foo~~ form works

*) economical when construction long span element

Limitations:-

* used of high strength materlous is costy
* Required skilled members
* Need of additional equipment cost of auxilloay equipment for tendency
* Need for quality control & inspection

Types of prestress:- / Tensioning devices:-

prestressing of concrete can be classified into several ways:

1. mechanical
2. Hydraulic
3. Electrical (thermal)
4. Chemical

The "mechanical" devices generally used include weights with (or) without lever transmission, ~~genera~~ geared transmission in conjuction with pulley blocks screw Jacks with (or) with out gear ~~devices~~ drives & wire-winding machines. These produced on a mass scale in factories.

"Hydraulic" Jacks being the simplest means of producing large prestressing forces, are extensively used as tensioning devices. Several commonly used patent Jacks are due to Freyssinet, magnel, Gifford udall & Baur Leonhardt for the range of 5-100 tonnes. It is important that during the tensioning operation the force applied should be accurately measured. In most of Jacks, calibrated pressure guages directly indicate the magnitude of force developed during the tensioning of the wires.

"Electrical" devices have been successfully used in erstwhile USSR since 1958 for tensioning of steel wires and deformed bars. The steel wires are electrically heated and anchored before placing concrete in the moulds. This method is often referred to as 'thermo electric prestressing'.

In "chemical" method, expanding cements are used and the degree of expansion is controlled by varying the curing conditions. since the expansion action of cement while setting is restrained, it induces tensile force in tendons & compressive stresses in concrete.

external pre-stressing:

when pre-stressing is acheived by elements located outside of concrete it is called external prestressing. The tendon can tie out side the member (or) inside the hollowspace of a box girders. This technique is adopted in bridges and strengthing of buildings & arches.

Internal prestressing:-

when the prestressing is achieved by elements located insidee the concrete it is called internal prestressing. most of the applications of pre-stressing are internal prestressing.

Pre tensioning:-

A method of prestressing concrete in which the tendons are tensioned before the concrete is placed. In this method, the prestress is imparted to concrete by bond b/n steel & concrete.

post tensioning:-

A method of ~~post~~ prestressing concrete by tensioning the tendons against hardened concrete. In this method, the prestress is imported to concrete by bearing.

Linear prestressing:-

when the prestressing members are straight (or), in the direction of pre-stressing the prestressing is called linear prestressing

Ex: Prestressing of beams, poles & slabs

circular prestressing:-

when the pre-stressed members are curv in the direction of prestressing the pre-stressing is called circular prestressing.

Ex:- Tanks, pipes & ~~silos~~ silos.

uniaxial prestressing:-

when the prestressing tendons are $\parallel^{el}$ to one-axis it is called uni-axial pre-stressing.

eg:- longitudnal pre-stressing of beams.

Bi-axial prestressing:-

when the prestressing tendons are $\perp^{er}$ to 2 axis it is called Bi-axial prestressing

eg:- slabs.

Multi-axial prestressing:-

when the pre-stressing tendons are $\parallel^{el}$ to more than 2-axis it is called multi axial pre-stressing.

eg: Domes.

Full prestressing:-

when the level of pre-stressing is such that no tensile-stress is allowed in concrete u service loads. It is called fully pre-stress.

Limited (or) partial prestressing:-

when the level of prestressing is such the under tensile stresses due to the service loads the crack width is with in the allowable limit is called limited (or) partial pre-stressing.

METHODS OF PRE-STRESSING:-

The prestressing system and devices are describe in the ~~fo~~ following 2-types.

* pre-tensioning
* post-tensioning.

Bricksoft

pre-tensioning:-

It is the application before casting to apply tensile force to high tensile steel tendon around which concrete has to classified.

stages:- In ~~prestr~~ pretensioning systems the high strength steel tendon are parallel ~~to~~ b/n 2 end abutments prior to concrete casting. The abutment are fixed at the ends of pre-stressed bed.

Once the concrete attain the desired strength for pre-stressing the tendon are cut from the abut-ments

The prestress is transfer to the concrete from the tendons due to bond b/n concrete and tendon. During the transfer of pre-stress the member undergoes elastic shortening. If the tendons are located ecentrically the member is likely to bend and deflect upwards (camber)

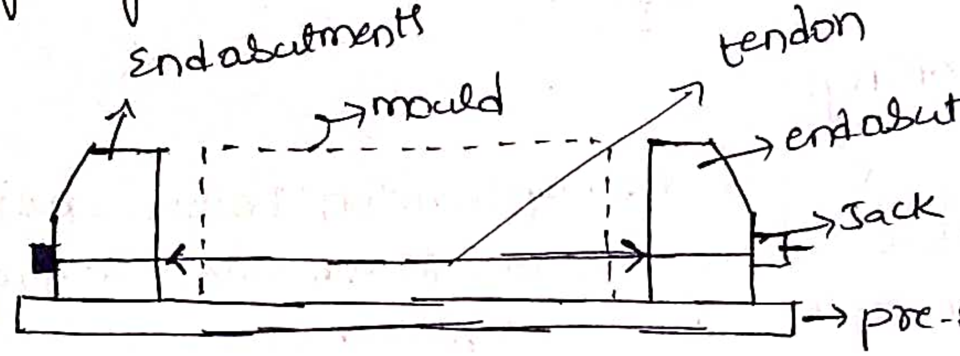
The various stages of pre-stressing are summerised as follows:

→ Anchorage of tendons against the end abutments
→ placing of Jacks
→ Applying tension to the tendons
→ casting of concrete
→ curing of concrete

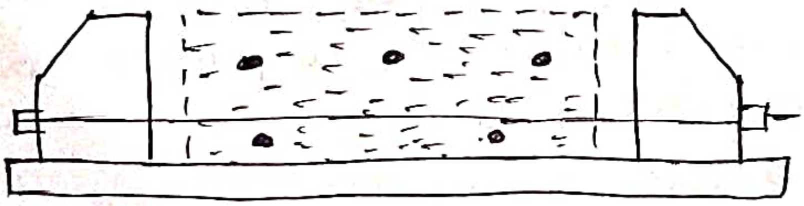
→ cutting of tendons.

During the transfer of pre-stress due to elas. shortening camber will developed in the member

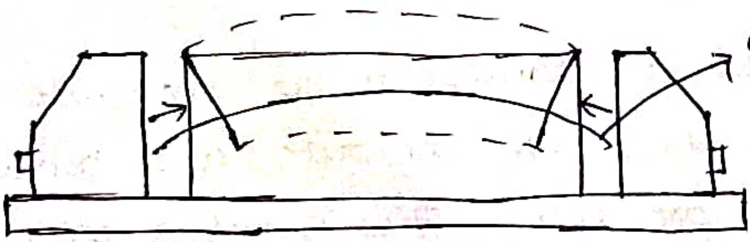
The stages are shown in schematically in the following figures.



a) Applying tension to the tendon



b) casting of concrete



Advantages:

→ It is suitable for pre-cast members in bulk
→ In pre-tensioning large anchorage devices is not present

Disadvantages:

→ A pre-stressed bed is required for the pre-tension operation
→ There is a waiting period in the pre-stressing bed before the concrete attains sufficient strength

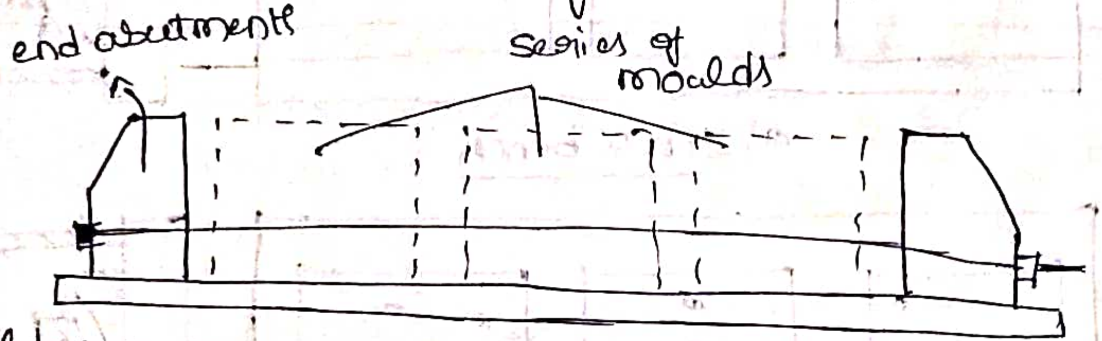
Devices: The essential devices for pre-tensioning are as follows.

1. prestressing bed
2. end abutments
3. mould (or) shuttering
4. Jacks
5. Anchoring devices
6. Harphing devices (optional)

Prestressing bed and end abutments:-

Hoyes-system / long line system.

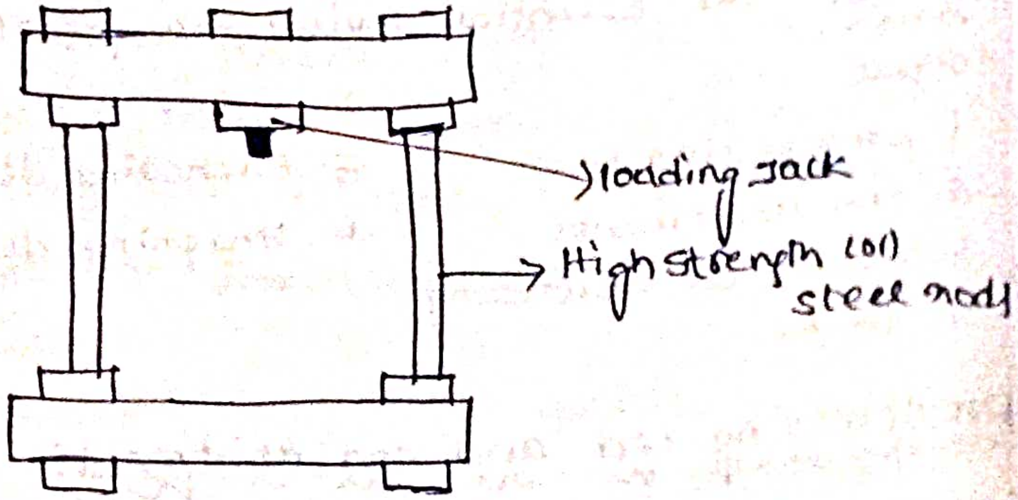
An extension of previous system is the "hoyes system". This system is generally used for mass production. The end abutment are kept sufficient distance apart and several members are cast in a single line. shuttering is provided at the sides and b/n the members. This system is also called long-line method.



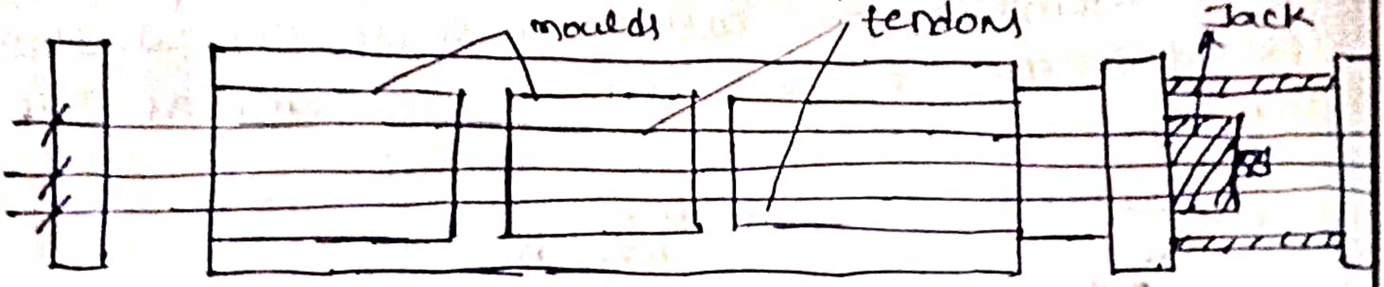
mould (or) shuttering:-

The abutments should be sufficiently stiff and should have good foundations. This is usually an expansive proposition. particularly when large pre-casting force is required.

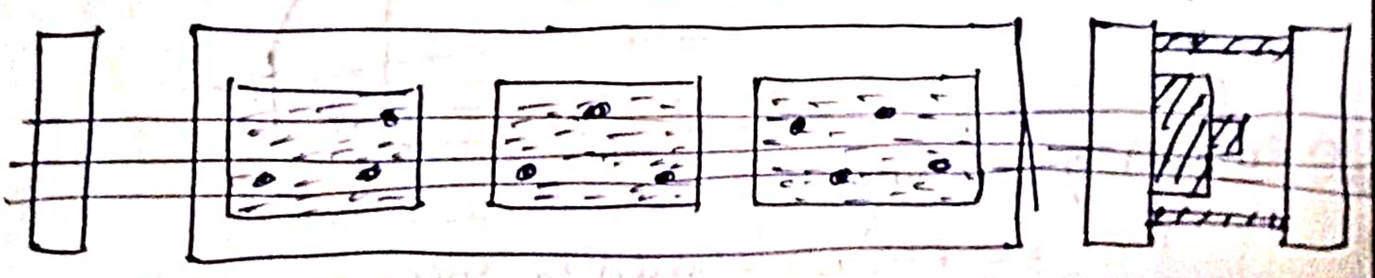
It is possible to avoid transmitting the heavy loads o foundation by adopting self-equilibrium system. Typical n laboratory systems this is done by tension frame.



The frame is generally adopted for a pre-tensi[oning] called "stress bench". The concrete mould is placed with in the frame and the tendons are stretched and anchored on the booms of the frame.



a) stress bench



b) stress bench after casting of concrete

Jacks:

The Jacks are used to apply tensions to the tend[ons]. "Hydraulic Jacks" are commonly used. These Jacks wor[k] on oil pressure generated by a pump.

The principle behind them is pascal law. The loa[d] applied by a Jack is measured by the pressure lo reading from a gauge attached to the load cell

Anchoring devices:- Anchoring devices are often made on the wedge and friction principle. In pre tensioned members the tendons are to be held in tension during the casting & hardening of concrete.

Harping devices:- The tendons are frequently bent except incase of slabs, poles. The tendon are bent in b/n the supports with a shallow sag.

post-tensioning:

In post-tensioning the tension is applied to the tendons after hardening of the concrete.

Stages:- The stages of post-tensioning are

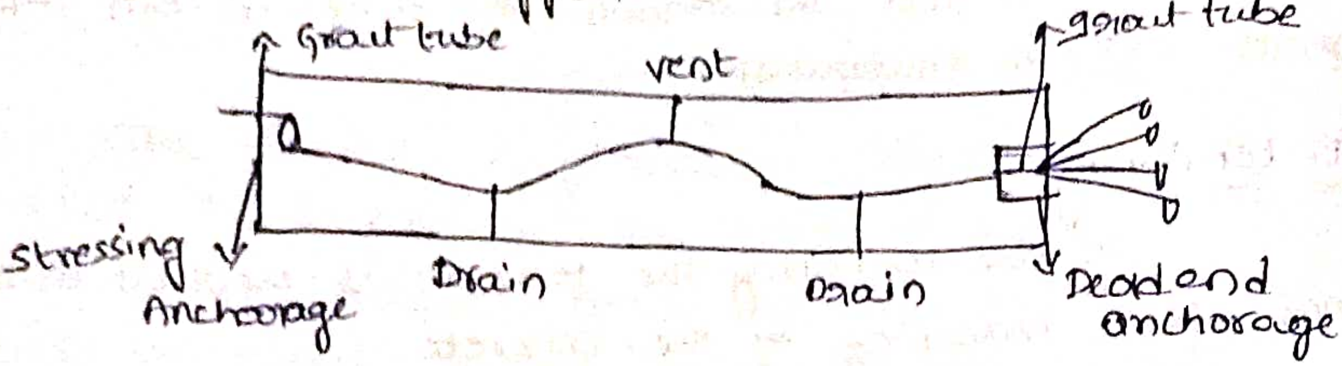
1. casting of concrete
2. placing of tendons
3. placement of anchorage block & Jack
4. Applying tension to tendon
5. seating of wedges
6. Cutting of tendons.

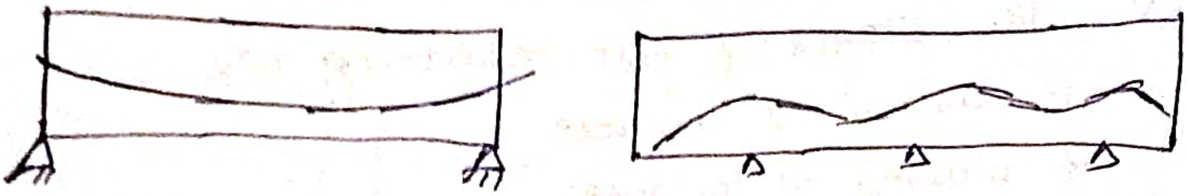
In post tensioning systems the ducts for the tendons or) strands are placed with the r/f before casting of concrete. The tendons are placed in the ducts after casting of concrete. The ducts prevents contact b/n concrete & tendon during the tension operation.

If the ducts are filled with grout then it is as bonded post-tensioning. The grout is heated with cement (or) sand cement mortar containing suitable admixture.

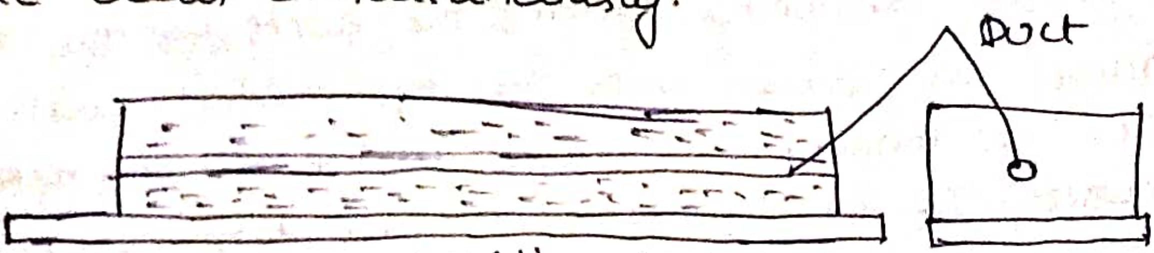
In unbonded post tensioning, as the name suggest the ducts are never grouted and the tendon is held in tension by the end anchorages.

The following sketches show a schematic represen-tation of grouted post-tensioned member. The profile of duct depends on the support conditions. For a S.S member, the duct has a sagging profile b/n the ends. For continuous member the duct sag in the sp & hog over the support

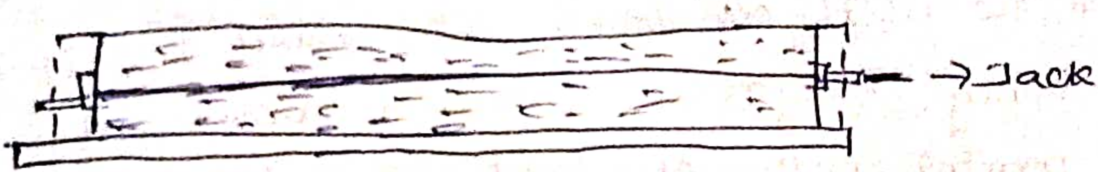




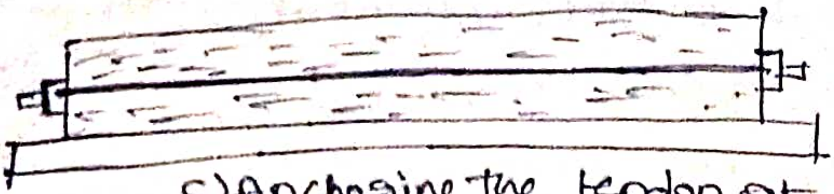
The stages are shown schematically in the following fig. After anchoring a tendon at one end, the tension is applied at the other end by Jack. The tensioning of tendons and pre-compression of concrete occur simultaneously.



a) casting bed



b) Tensioning of tendons



c) Anchoring the tendon at the streching end.

Advantages:-

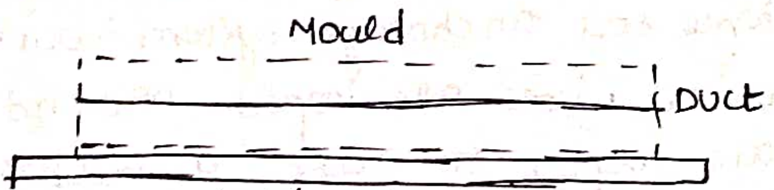
* post tensioning is suitable for heavy cast in place members
* The waiting period in the casting bed is less
* The transfer of pre-stress is independent of transmission length.

Disadvantages:-

* requirement of anchorage device and grouting equipment.

Devices:- The Essential devices for post-tensioning are

1. casting bed
2. mould / shuttering
3. Ducts
4. Anchoring devices
5. Jacks
6. couplers (optional)
7. Grouting Equipment (optional)



Anchoring devices:-

Anchoring devices members the transfer the pre-stress to concrete. The devices are based on the following principles of anchoring the tendons.

1. wedge action
2. Direct bearing
3. Looping of wires.

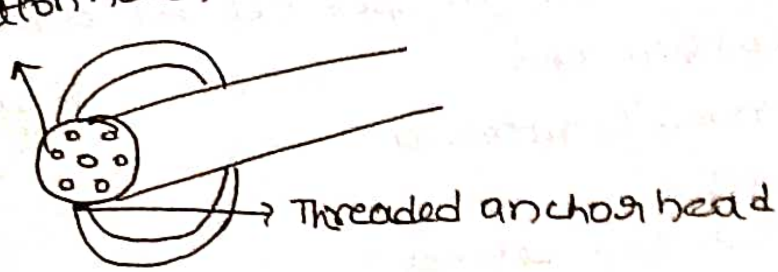
wedge action:-

The anchoring device based on wedge action consist of an anchorage block & wedges. The strands are held by frictional grip of the wedges.

in the anchorage block. some examples of systems based on the wedge-action are freyssinet, Gifford-Udall, Anderson & magnet-Balton anchorages.

Direct bearing:

The rivet (or) bolts heads (or) buttons formed the end of the wires directly bear against a block. The BBRV post-tensioning system and prescon system are based on this principle. The following fig shows anchoring.



Looping of wires:-

The Baur-Leonhardt system, Leoba system and a the dwidag single bar anchorage system work on this principle where the wires are looped around the concrete. The wires are looped to make a bulb.

Couplers:- The couplers are used to connect strands (or) bars. They are located at the Junction of the members for example at (or) near columns in post-tensioned slabs on piers in post-tensioned bridge decks.

Grouting:- Grouting can be defined as filling of duct, with a material that provides an anticorrosive alkaline.

The major part of grout comprises of water & cement, with a water-to-cement ratio of about 0.5 together with some water-reducing admixture expansion agent.

Principles of prestressing:-

→ It is a method in which compression force is applied to the reinforced concrete.

→ In order to reduce the tensile stresses ie (cracks)

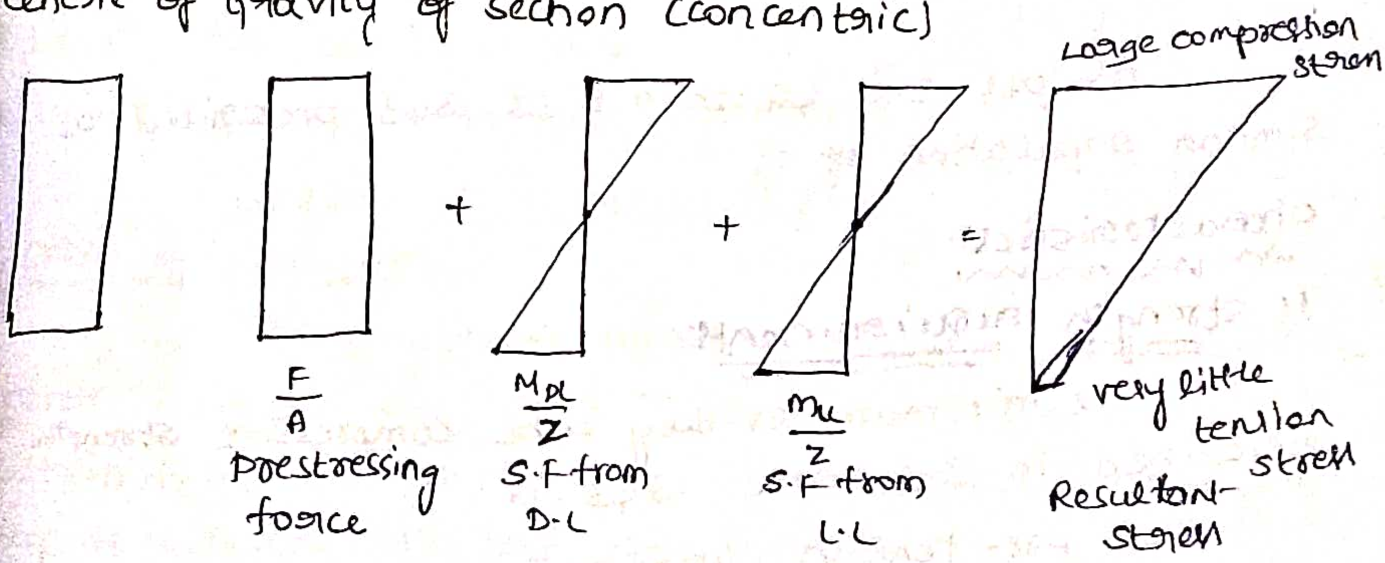
→ It is treated as elastic material

→ The concrete can be visualized to two compressive forces.

* internal prestressing force (Direct stress)
* External " " (Due to D.L & L.L)

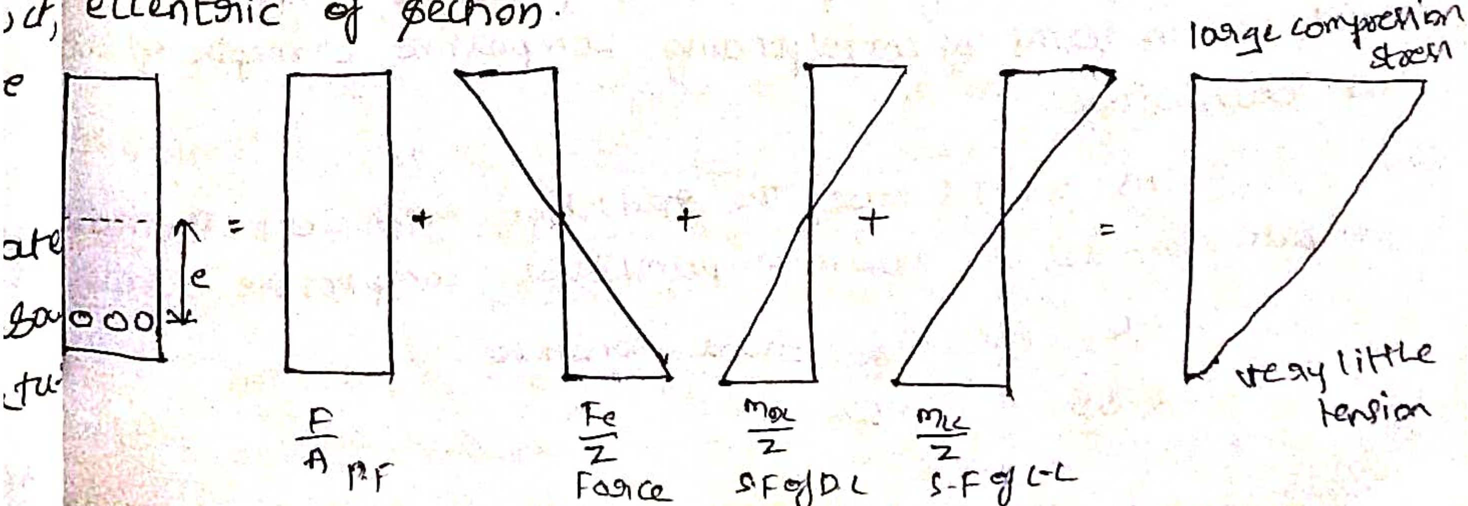
→ It is used for a combination of high strength concrete & steel

→ stress in concrete when pre-stressing is applied of the centre of gravity of section (concentric)



(a) Apply stress into the tendons (I-se)
(b) Transfer stress into the members (T-sec)
(c) Apply loads to members (service stage)

→ stress in concrete when prestressing is applied at the eccentric of section.



Applications:-

The prestressed concrete applied mostly in

→ Dome shaped structures → Railway sleepers
→ slabs in buildings → Nuclear power plants
→ off shore platform
→ Thermal plants

Materials used for prestressed concrete:-

(High strength concrete & steel)

→ prestressed concrete rises high concrete strength & high tensile strength compare to R.C member in order to reduce low shrinkage, creep & increases the modulus elasticity.

As per I.S 456:2000 & IS: 1343 prescribed a similar stipulation of 5%.

Characteristics:-

1. Strength requirements:-

The minimum 28-day cube compressive strength prescribed in I.S code 1343 is

For pre-tension : $40 N/mm^2$
for post-tension : $30 N/mm^2$

2. Permissible stress in concrete:-

The permissible compressive and tensile stress concrete at the stage of transfer & service loads are defined in terms of corresponding compressive strength of co at each stage.

As per I.S code, The reduction coefficient applied compute the design maximum permissable compressive stress

0.41 for m_{30} grade concrete
0.35 " m_{60} " "

3 Shrinkage of concrete:

The shrinkage of concrete in prestressed members is due to the gradual loss of moisture which results in change in volume. The drying shrinkage depends on the aggregate type & quantity, relative humidity, w/c ratio in mix & the time of exposure. It also depends upon the degree of hardening of the concrete at the commencement of drying.

The commonly used aggregates in increasing order of effectiveness in restraining shrinkage, are sand stone, basalt gravel, granite, quartz & lime stone.

It is formed due to losses of moisture concrete

3×10^{-4} for pre-tensioned.

$2\times10^{-4}/\log(t+2)$ for post-tensioned.

where, $t \rightarrow$ age in days.

creep of concrete:

The progressive inelastic strains due to creep in a concrete member are likely to occur under the smallest sustained stresses at ambient temperature. Shrinkage & creep of concrete are basically similar

55% of 20 years

	P.C	R.C
	3 month	1 months
→ Rate 75% of 50 year	1 year	3 months

High-tensile steel:-

For P.C members, the high-tensile steel used generally consist of wires, bars (or) strands. The higher tensile strength is acheived by marginally increasing the carbon content in steel comparision with mild steel.

High-tensile steel usually contains 0.6-0.85% carbon, 0.7-1% manganese, 0.05% of sulphur & phosphorous with

with traces of silicon.

The high-carbon steel ingots are hot-rolled into rods & cold-drawn through a series of dies to reduce the diameter and increase the tensile strength. The proce of cold-drawing through dies decrease the ductility of the wires. normal sizes are 2.5, 3, 4, 5, 7 & 8 mm φ & they sho conform to I.S codes

The hard-drawn steel wires which are indented are preferred for pretensioned elements because of their sup bond characters. 2 to 5mm (small φ)

The high-tensile steel bars commonly employed in prestressing in nominal sizes of 10, 12, 16, 20, 22, 25, 28 & 32 φ

Strength requirements:-

The ultimate tensile strength of a plain hard-d steel wire varies with its diameter. The tensile strength dec with increase in diameter of the wires.

Nominal φ	Tensile strength (N/mm²) (minimum)
2.5	2010
3.0	1865
4.0	1715
5.0	1570
7.0	1470
8.0	1375

4. DESIGN FOR FLEXURE

When prestressed concrete members are subjected to different bending loads, different types of flexure failures are possible at critical sections. Depending upon

* % of r/f in the section
* Bond b/n tendons & concrete
* compressive strength of concrete &
* ultimate tensile strength of tendons.

Types of flexural failures:-

1. Fracture of steel in Tension:-

This mode of failure occur in the beam when the tension zone is provided with least amount of steel reinforcement. If the concrete in tension zone gets cracked will lead to develop extra tensile stresses which are unable to with stand by the steel provided in the tensile zone, & cause of failure of prestressed concrete beam.

* This failures occurs sudden with out warning
* In order to prevent failure a minimum steel r/f provided in c/s IS-1343 recomended a min. r/f of 0.15-0.2% of c/s area in P.S.C

2. Failure of under-reinforced sections:-

The beam observes excess elongation of steel along with crushing of concrete. This is because of large amount of steel is provided in the compression zone (greater than min steel r/f in tension zone) - Due to this, at the neutral axis near the compression face gets increased followed by increased bending loads at critical section of beam.

→ Large deflections & wide cracks are observed at ~~on~~ compression face.

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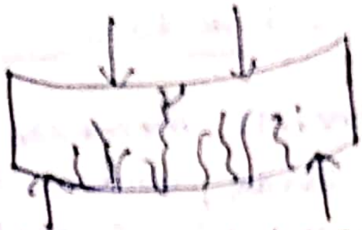
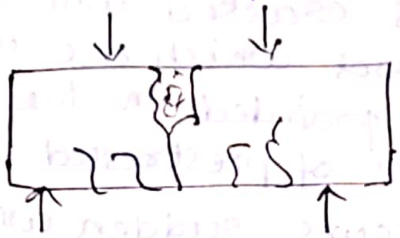


Failure of under ~~r~~ reinforced section

3. Failure of over reinforced section:-

Over reinforced section failure occures due to crushing of concrete. In this section observes small deflections & narrow cracks. In this the % of r/f steel due to which the compressive strength of concrete & tensile strength of steel are increased

* Failures are sudden & occur without any warnings
* Amount of steel provided in this section should not be greater than steel ~~pr~~ required for Balancing section.



over r/f section.

→ Some other failures occurs when transverse shear failure & web crippling are caused due to improper design of member in shear & providing webs in the section.

Assumptions:-

* Tensile strength of concrete is ignored.
* After Bending the plane sections will be normal to the axis
* The strain in bonded r/f whether in tension (or) compression is same.

Strain compatibility method:-

The method by which the flexural strength of prestressed concrete is estimated based on the compatability of strain.

Applied:-

→ Distribution of concrete in a plane section remain plane even after bending.

→ In tension members, the resistance of concrete is neglected

→ Max strain at failure attains a particular value.

Steps to determine flexural strength:-

1. After extracting the losses from stress-strain curve for steel calculate the effective strain "e_s" in steel.
2. Calculate trail value "x_u" from N-A (assumed) & know ($e_{su} - e_{sc}$) [∵ Adopt ultimate strain $e_{cu} = 0.0035$]
3. evaluate stress in steel "f_b"
4. Determine total compressive & tensile force from eq'n

compressive force $C_u = K_1 \times f_{ck} \times b \times x_u$

tensile " $T_u = A_p \times f_b$

where, A_p = Area of prestressing

K_1 = characteristic ratio

5. If $C_u = T_u$ then depth of N-A "x_u" is correct if not increase (or) decrease the depth
6. Determine ultimate Moment

$$M_u = A_p f_b (d - K_2 x_u)$$

where, K_2 : characteristic ratio

7. Repeat two to three trails in order to obtain equal force.

I.S codal provisions:-

The indian standard code method (IS : 1343 1980) for computing flexural strength of $\square^{lar}$ sections (or) T-sections in which Neutral axis lies within the flange is based on the $\square^{lar}$ & Parabolic stress block as shown in figure.

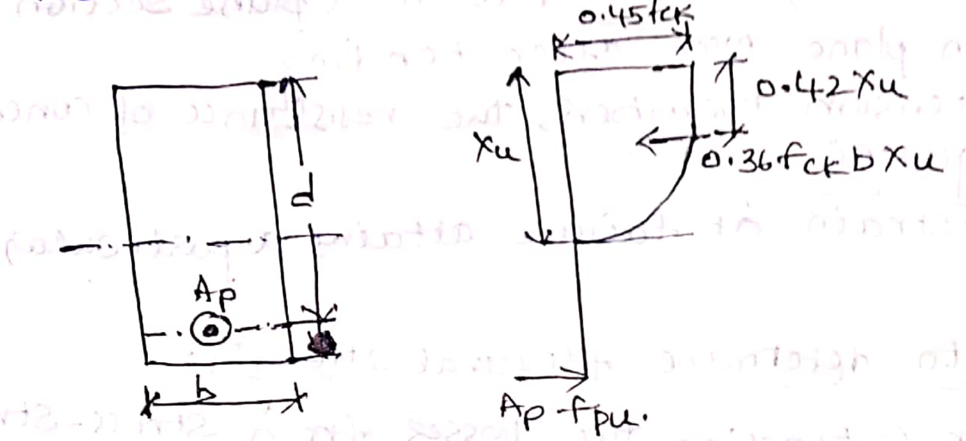


Fig:- Moment of Resistance for Rectangular section

The moment of resistance is obtained by

$$\{ M_u = A_p f_{pu} (d - 0.42 x_u) \}$$ [Pg: 59 code 800 F]

where, M_u = ultimate moment of resistance of the section

f_{pu} = tensile stress developed in tendons at the failure stage

f_p = characteristic tensile steel strength

A_p = Area of prestressing tendons

d = effective depth

x_u = Neutral-axis depth

The value of f_{pu} depends upon eff r_F ratio

$$= \left[\frac{A_p f_p}{b \cdot d f_{ck}} \right]$$

→ The values of f_{pu}, x_u are given in Table 11 (IS 1343) pg 59

If A_{pw} = Area of prestressing steel for web
A_{pf} = " " " " for flange
D_f = Thickness of flange

Then,
$$A_p = (A_{pw} + A_{pf}) \rightarrow (1)$$
But,
$$A_{pf} = 0.45 f_{ck} (b - b_w)\left(\frac{D_f}{f_p}\right) \rightarrow (2)$$
from solving eq(2) we get
$$\{A_{pw} = A_p - A_{pf}\}$$
For the eff ratio of $(A_{pw} f_p / b_w d f_{ck})$, the corresponding values of $(f_{pu}/0.87 f_p)$ & (x_u/d) are obtained from IS code

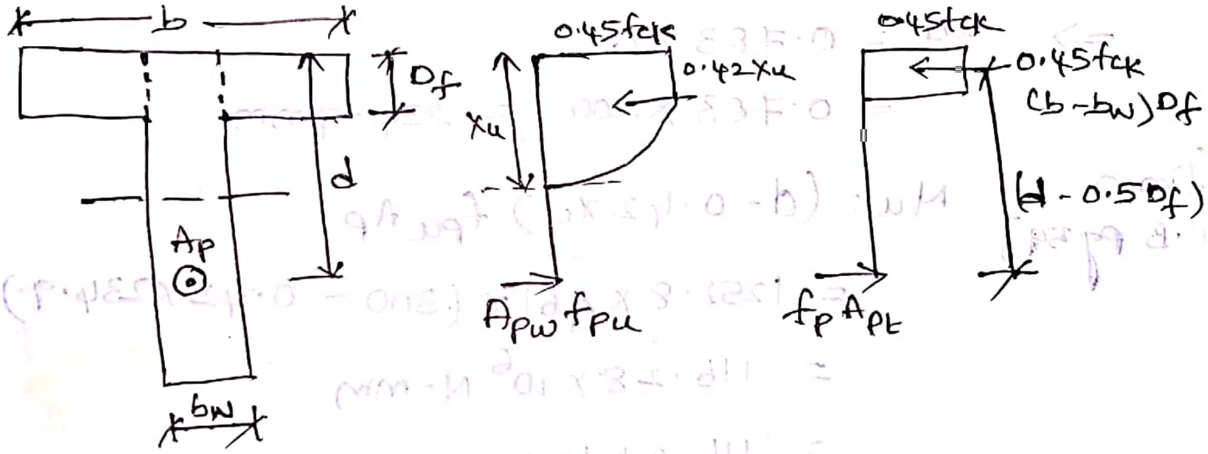


Fig: Moment of resistance of flanged sections ($x_u > D_f$)

$$\{M_u = f_{pu} \cdot A_{pw} (d - 0.42 x_u) + 0.45 f_{ck} (b - b_w) D_f (d - 0.5 D_f)\}$$

P) A pre tensioned prestressed concrete beam having a rectangular section, 150mm wide & 350mm deep, has an effective cover of 50mm. If $f_{ck} = 40\ N/mm^2$, $f_p = 1600\ N/mm^2$, & area of prestressing steel $A_p = 461\ mm^2$ calculate the ultimate flexural strength of the section using IS 1343 codal provisions.

Sol:- Given, $f_{ck} = 40\ N/mm^2$ $\quad b = 150\ mm$

$f_p = 1600\ N/mm^2$ $\quad d = 300\ mm$

$A_p = 461\ mm^2$

The effective q/F ratio is given by

$$\left(\frac{f_p A_p}{f_{ck} b d}\right) = \frac{1600 \times 461}{40 \times 150 \times 300}$$

$$= \frac{737{,}600}{1{,}800{,}000} = \underline{0.40}$$

From Table 11, of IS 1343 we know

$$\left(\frac{f_{pu}}{0.87 f_p}\right) = 0.9 \; ; \; \frac{x_u}{d} = 0.783$$

$$\Rightarrow f_{pu} = 0.9 \times 0.87 \times f_p$$

$$= 0.9 \times 0.87 \times 1600 = 1252.8 \, N/mm^2$$

$$\Rightarrow x_u = 0.783 \times d$$

$$= 0.783 \times 300 = 234.9 \, mm$$

[from C.B pg 59]

$$M_u = (d - 0.42 x_u) f_{pu} A_p$$

$$= 1252.8 \times 461 \times (300 - 0.42 \times 234.9)$$

$$= 116.28 \times 10^6 \, N\text{-}mm$$

$$= \underline{116.28 \, KN\,M}$$

P) A pretensioned T-section has a flange which is 300 mm wide 200 mm thick. The rib is 150 mm wide by 350 mm deep. The effective depth of the cross-section is 500 mm. Given, $A_p = 200 \, mm^2$, $f_{ck} = 50 \, N/mm^2$ & $f_p = 1600 \, N/mm^2$, estimate the ultimate moment capacity of the T-section using the Indian standard code regulations.

Sol:- Given, $f_{ck} = 50 \, N/mm^2$

$f_p = 1600 \, N/mm^2$

$A_p = 200 \, mm^2$

$b = 300 \, mm$

$d = 500 \, mm$

Assuming that the N-A falls within the flange, the value of $b = 300$ mm for computation of effective r/F ratio

$$\left(\frac{f_p A_p}{f_{ck} bd}\right) = \left(\frac{1600 \times 200}{50 \times 300 \times 500}\right)$$
$$= 0.04$$

from table 11, Pg : 59

$$\left(\frac{f_{pu}}{0.87 f_p}\right) = 1.0 \quad \& \quad \frac{x_u}{d} = 0.09$$

$$\Rightarrow \quad f_{pu} = 1 \times 0.87 \times 1600 = 1392 \ N/mm^2$$
$$x_u = 0.09 \times 500 = 45 \ mm$$

The assumption that the N-A falls with in the flange

$$\therefore M_u = f_{pu} \cdot A_p (d - 0.42 x_u)$$
$$= 1392 \times 200 (500 - 0.42 \times 45)$$
$$= 133.9 \times 10^6 \ N\text{-}mm$$
$$= 134 \ KNm$$

*) A pretensioned, T-sec has a flange 1200mm wide & 150 mm thick. The width & depth of the rib are 300 & 1500 mm respectively. The high tensile steel has an area of 4700mm² & is located at an eff depth of 1600 mm. If the characteristic strength of the concrete & the tensile strength of steel are 40 & 1600 N/mm² resp. calculate flexural strength of the T-section

Sol: Given, $A_p = 4700 mm^2$ $\quad d = 1600mm$
$f_{ck} = 40N/mm^2$ $\quad D_f = 150mm$
$b = 1200mm$
$b_w = 300mm$

→ Area of pre-stressing tendons $(A_p) = (A_{pw} + A_{pf})$

$$A_{pf} = 0.45 f_{ck} (b - b_w)\left(\frac{D_f}{f_p}\right)$$
$$= 0.45 \times 40 \times (1200 - 300) \times (150/1600)$$

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$= 1518.75 \text{ mm}^2$

$\therefore A_{pw} = A_p - A_{pf}$

$= 4700 - 1518.75$

$= 3181.25 \text{ mm}^2$

effective r/f ratio $= \dfrac{A_{pw} f_p}{b_w d f_{ck}}$

$= \dfrac{3181.25 \times 1600}{300 \times 1600 \times 40}$

$= 0.265$

from Table 11 of IS: 1343

$\dfrac{f_{pu}}{0.87 f_p} = 1.00$

$\Rightarrow f_{pu} = 0.87 \times f_p$

$= 0.87 \times 1600 = 1392 \text{ N/mm}^2$

& $\dfrac{x_u}{d} = 0.56 \Rightarrow x_u = 0.56 \times d$

$= 0.56 \times 1600$

$= 896 \text{mm}$

$\therefore$ Moment of Resistance $(M_u) =$

$f_{pu} \cdot A_{pw}(d - 0.42 x_u) + 0.45 f_{ck}(b - b_w) D_f (d - 0.5 D_f)$

$= 1392 \times 3181.25 (1600 - 0.42 \times 896) + 0.45 \times 40 \times (1200 - 300) \times 150 (1600 - 0.5 \times 150)$

$= 5.418 \times 10^9 + 3.705 \times 10^9$

$= 9.123 \times 10^9$

$= 9123 \times 10^6 \text{ N-mm} \cong 9123 \text{ kN-m}$

*) A post-tensioned beam with unbonded tendons is of rectangular section 400 mm wide with an effective depth of 800mm. The c/s area of pre-stressing steel is 2840 mm². The eff. prestress in steel after all losses is 900 N/mm². The eff. span of beam is 16m. If $f_{ck} = 40 \text{ N/mm}^2$, estimate

the ultimate moment of resistance of the section using IS:1343 code

Sol: $f_{ck} = 40\ N/mm^2$ $\quad b = 400$

$f_{pe} = 900\ N/mm^2$ $\quad d = 800$

$A_p = 2840\ mm^2$ $\quad L = 16m$

Span ratio $= L/d = \frac{16000}{800} = 20.$

The effective q_F ratio is given by

$$\left(\frac{A_p \cdot f_{pe}}{b \times d \times f_{ck}}\right) = \frac{900 \times 2840}{400 \times 800 \times 40}$$
$$= 0.2$$

From Table of IS:1343 – (unbonded tendons)

$\frac{f_{pu}}{f_{pe}} = 1.16$; $\frac{x_u}{d} = 0.58$

$\Rightarrow f_{pu} = 1.16 \times 900$ $\quad x_u = 0.58 \times 800$

$= 1044\ N/mm^2$ $\quad = 464 mm$

$\therefore$ ultimate moment of resistance

$$M_u = A_p(d - 0.42 x_u) \cdot f_{pu}$$
$$= 2840(800 - 0.42 \times 464) \cdot 1044$$
$$= 1794 \times 10^6\ N\text{-}mm$$
$$= 1794\ KN\text{-}m$$

*). A post-tensioned bridge girder with unbonded tendons is of box section of overall dimensions 1200 mm wide by 1800 mm deep, with wall thickness 150 mm. The high-tensile steel has an area of 4000 mm^2 & is located at an effective depth of 1600 mm. The effective prestress in steel after all losses is 1000 N/mm^2 & the effective span of girder is 24m. If $f_{ck} = 40\ N/mm^2$ & $f_p = 1600\ N/mm^2$, estimate the ultimate flexural strength of the section.

Sol:- Given, $A_p = 4000 mm^2$ $b = 1200mm$

$f_{ck} = 40 N/mm^2$ $b_w = 300mm$

$f_p = 1600 N/mm^2$ $d = 1600mm$

$f_{pe} = 1000 N/mm^2$ $D_f = 150mm$

$L = 24m$

eff. span ratio $L/d = \frac{24000}{1600} = 15$

∴ Area of prestressing tendons $(A_p) = A_{pw} + A_{pf}$

$$\therefore A_{pf} = 0.45 f_{ck} (b - b_w) \left(\frac{D_f}{f_p}\right)$$

$$= 0.45 \times 40 \times (1200 - 300) \left(\frac{150}{1600}\right)$$

$$= 1518.75 mm^2$$

$$A_{pw} = A_p - A_{pf}$$

$$= 4000 - 1518.75 = 2481.25 mm^2$$

Thus, $\frac{A_{pw} f_{pe}}{b_w d f_{ck}} = \frac{2481.25 \times 1000}{3000 \times 1600 \times 40} = 0.0129$

from Table 11, of IS 1343 (un Bonded Tendons)

$\frac{f_{pu}}{f_{pe}} = \cancel{1.430}\ 1.31$; $\frac{x_u}{d} = 0.41$

$\Rightarrow f_{pu} = 1.31 \times 1000$; $x_u = 0.41 \times 1600$

$= 1310 N/mm^2$ $= 656 mm$

$$M_u = f_{pu} A_{pw} (d - 0.42 x_u) + 0.45 f_{ck} (b - b_w) \cdot D_f (d - 0.5 D_f)$$

$$= 1310 \times 2481.25 (1600 - 0.42 \times 656) + 0.45 \times 40 (1200 - 300) \cdot 150 (1600 - 0.5 \times 150)$$

$$= 4.305 \times 10^9 + 3.705 \times 10^9$$

$$= 8.01 \times 10^9 \Rightarrow 8010 \times 10^6 N\text{-}mm$$

$$M_u = 8010 KN\text{-}M$$

UNIT :- 5

SHEAR AND TORSIONAL RESISTANCE OF PRE-STRESSED CONCRETE MEMBERS

Shear & principal stresses:-

The shear distribution in an uncracked members of structural concrete for which the deformation is assumed to be linear which is the function of shear force & properties of the cross-section of member. Thus, shear stress at a point is expressed as

$$\left\{ \tau_v = \frac{VS}{Ib} \right\}$$

where, τ_v = shear stress

V – Shear force

S – First moment of area

I – Moment of Inertia

b – width of the section

In pure shear, the strength of concrete is twice that of the strength in tension. Hence, cracks are observed at the points of development of maximum shear stress diagonally. The effect of this Max. shear stress, (τ_v) also produces principal tensile stresses on diagonal plane.

The major & minor principal stresses produced on diagonal plane is given by eq'n

$$f_{\substack{max \\ min}} = \left[\left(\frac{f_x + f_y}{2} \right) \pm \frac{1}{2} \sqrt{(f_x - f_y)^2 + 4\tau_v^2} \right)$$

where, f_x, f_y → vertical & horizontal directional stresses respectively

If the direct stress are compressive, then the magnitude of principal stresses in prestressed concrete member, under the working loads the principal stresses have to be a compressive in nature in order to eliminate cracks in diagonaley formed in concrete.

The strength of shear resistance is improved by prestressing the concrete member horizontaley, verticaley & by sloping cable (inclined)

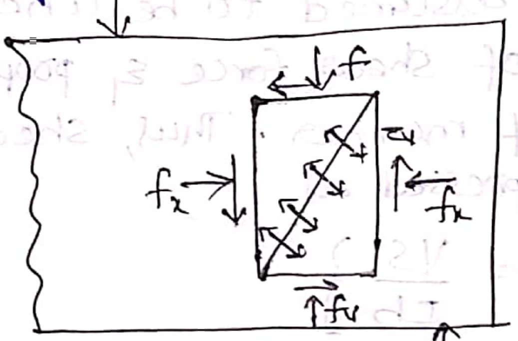
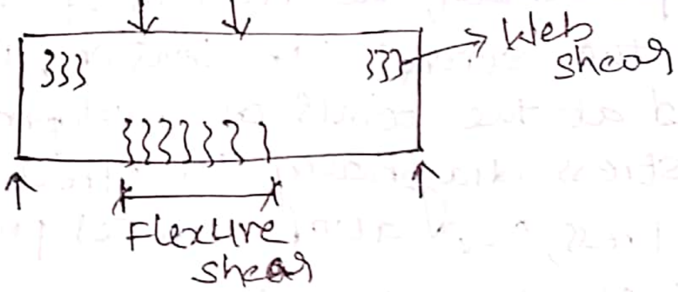


Fig:- principale tensile stresses in a prestressed member

Types of shear cracks:-

* web shear cracks
* flexure shear cracks



P) A prestressed concrete beam (span = 10m) of □ section 120mm wide & 300mm deep, is axially prestressed by a cable carrying an effective force of 180 KN. The beam supports a total UDL of 5 KN/m which includes the self-weight of the member. compare the magnitude of the principal tension developed in the beam with and without the axial prestress.

Sol :- Given, $l = 10\,mt$
$= 10,000\,mm$

$b = 120\,mm$; $d = 300\,mm$

Force $= 180\,KN$

Moment of inertia $(I) = \frac{bd^3}{12} = \frac{120 \times 300^3}{12}$
$= 270 \times 10^6\,mm^4$

UDL Load $(w_d) = 5\,KN/m$

∴ Principal stresses of a beam

$$f_{\substack{max\\min}} = \left\{\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right\}$$

∴ Shear force at support, $V = \frac{wl}{2}$
$= \frac{5 \times 10}{2}$
$= 25\,KN$

Axial prestress, $f_x = \frac{\text{force}}{\text{Area}}$
$= \frac{180 \times 10^3}{120 \times 300}$
$= 5\,N/mm^2$

$f_y = 0$

Maximum shear stress at support, $\tau_v = \frac{3}{2}\tau_{Avg}$

$\Rightarrow \tau_v = 3/2 \cdot V/bh$
$= 3/2 \cdot \frac{25 \times 10^3}{120 \times 300}$
$= 1.05\,N/mm^2$

with axial prestressing $(f_x = 5 ; f_y = 0)$

$$f_{\substack{max\\min}} = \left\{\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right\}$$

$$= \left\{\left(\frac{5+0}{2}\right) \pm \frac{1}{2}\sqrt{(5-0)^2 + 4(1.05)^2}\right\}$$

$f_{\substack{max\\min}} = 5.207, -0.21\,N/mm^2$

without axial prestressing $(f_x = f_y = 0)$

$$f_{\substack{max \\ min}} = \left(0 \pm \frac{1}{2}\sqrt{0+4(1.05)^2}\right)$$

$$= \pm 1.04 \, N/mm^2$$

Hence, the axial prestress, the principal tension is reduced by

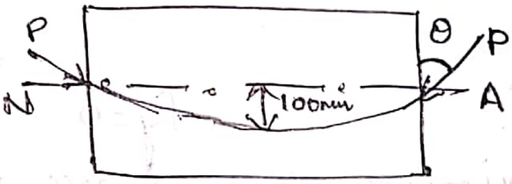
$$\Rightarrow \frac{1.04-0.21}{1.04} \times 100 = 79.8 \approx \underline{80\%}$$

P) For the beam above problem instead of axial prestressing a curved cable having an eccentricity of 100 mm at the centre of span & reducing zero at the support is used, the effective prestressing force in the cable is 180 KN. Estimate the % of reduction in the principal tension in compression with the case of axial prestressing?

Sol) Given,

$e = 100mm$

$P = 180 \, KN$



slope of the cable at support $\Big\}$ $\theta = \frac{4e}{L}$

$$= \frac{4 \times 100}{10000} = 0.04 \, rad$$

∴ ~~Net shear force at support~~

~~V_{net}~~

vertical component of prestressing force $\Big\} = P \times \theta$

$= 180 \times 0.04$

$= 7.2 \, KN$

∴ Net shear force at support

$V_{net} = 25 - 7.2$

$= 17.8 \, KN$

$\therefore$ Maximum shear stress $\tau_v = 3/2\, \tau_{Avg}$

$$= \frac{3}{2} \cdot V/bh$$

$$= \frac{3}{2} \times \frac{17.8 \times 10^3}{120 \times 300}$$

$$= 0.74\ N/mm^2$$

with axial prestressing compression

$$f_{max/min} = \left[\left(\frac{5+0}{2}\right) \pm \frac{1}{2}\sqrt{(5-0)^2 + 4(0.74)^2}\right]$$

$$= (2.5 \pm 2.61)$$

$$= (5.11, -0.1)\ N/mm^2$$

$$f_{max} = 5.11\ N/mm^2\ (comp)$$

$$f_{min} = -0.11\ N/mm^2\ (Tens)$$

In comparision with axial prestressing, the % reduction in principal tension

$$tension = \frac{0.21 - 0.11}{0.21} \times 100 = 47.6 \approx 48\%$$

$$compression = \frac{5.21 - 5.11}{5.21} \times 100 = 1.91 \approx 2\%$$

P) If the beam in above problem is additionally prestressed by vertical cables imparting a stress of $2.5\ N/mm^2$ in the direction of depth of beam, Estimate the nature of principal stress at the Support reaction.

Sol: Given, $f_y = 2.5\ N/mm^2$

$$f_x = \frac{P}{A} = \frac{180 \times 10^3}{120 \times 300} = 5\ N/mm^2$$

$$\tau_v = 0.74\ N/mm^2$$

$$f_{max/min} = \left[\left(\frac{5+2.5}{2}\right) \pm \frac{1}{2}\sqrt{(5-2.5)^2 + 4(0.74^2)}\right]$$

$$= (3.75 \pm 1.45)$$

$f_{max} = 5.2\ N/mm^2$ (comp); $f_{min} = 2.29\ N/mm^2$ (comp)

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P) A concrete beam having a $\square^{lar}$ section 150mm x 300mm deep is prestressed by a parabolic cable having an eccentricity of 100mm at centre of span reducing ~~at~~ zero at supports. The span of beam is 8m. The beam supports a live load of 2kN/m. Determine the effective force in the cable to balance the D.L & L.L on the beam. Estimate the principal stresses at support reactions.

Sol: Given, $b = 150mm$; $d = 300mm$; $e = 100mm$

$$l = 8mt = 8000mm$$

$$\text{Self weight} = g = 0.15 \times 0.3 \times 1 \times 24$$
$$= 1.08 \ N/mm \ (or) \ kN/m$$

$$\text{live load } (q) = 2kN/m$$

$$\therefore \text{total load } (\omega) = 2 + 1.08$$
$$= 3.08 \ kN/m$$

$$P = ?$$

From load balancing concept =

$$(\text{moment})_P = (\text{moment})_\omega$$

$$\Rightarrow P \cdot e = \frac{\omega l^2}{8}$$

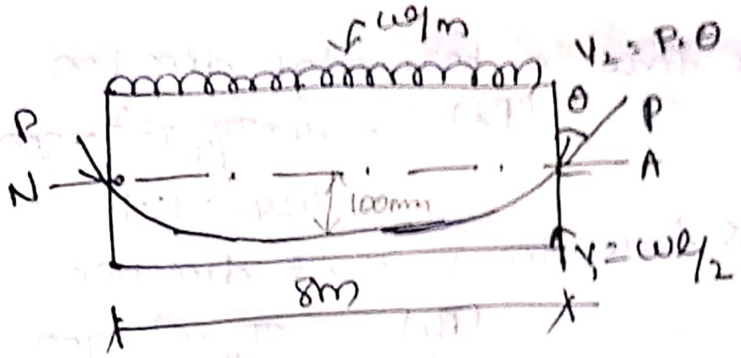
$$\Rightarrow P \cdot (100) = \frac{3.08 \times 8000^2}{8}$$

$$P = \cancel{264.4}\ 246.4 \times 10^3 \ N$$
$$= \cancel{264.4}\ 246.4 \ kN$$

$$\therefore \text{axial force of prestress } (f_x) = \frac{P}{A} = \frac{\cancel{264.4}\ 246.4 \times 10^3}{150 \times 300}$$

$$f_y = 0 \qquad\qquad = 5.5 \ N/mm^2$$

$$\text{Max. Shear stress } \tau_v = \frac{3}{2} \cdot \tau_{Avg}$$
$$= \frac{3}{2} \cdot V/bh$$
$$= \frac{3}{2} \times \frac{V}{150 \times 300}$$



$V_1 = \frac{wl}{2} = \frac{3.08 \times 8000}{2} = 12.3\,KN$ at sup $[x=0]$

$V_2 = P.\theta \Rightarrow \theta = \frac{4e}{l^2}(l-2x)$

$= \frac{4\times100}{l^2}(l)$

$= 246.4 \times 0.05$

$= 12.3\,KN$

$= \frac{4\times100}{l} = \frac{400}{8000} = 0.05$

$\therefore$ Net shear force $(V) = V_1 - V_2$

$= 12.3 - 12.3 = 0$ //

Shear stress $\tau_v = \frac{3}{2} \times 0$

$= \underline{\underline{0}}$

$\therefore f_{\substack{max \\ min}} = \left[\left(\frac{5.5}{2}\right) \pm \frac{1}{2}\sqrt{5.5^2 + 4(0)^2}\right]$

$= (2.75 \pm 2.75)$

$= (5.5, 0)$

$\therefore f_{max} = 5.5\,N/mm^2$; $f_{min} = \underline{\underline{0}}$

P) A prestressed I-sec has the following properties

Area $= 55 \times 10^3\,mm^2$

Second moment of area $= 189 \times 10^7\,mm^4$

Statistical moment $= 468 \times 10^4\,mm^3$

Thickness of web $= 50\,mm$

It is prestressed horizontally by 24 wires of 5mm φ & vertically by similar wires at 150 mm c/c. All the wires carry a tensile stress of $900\,N/mm^2$. calculate the principal stresses at the centroid when shearing force of 80KN acts upon this section

Sol: prestress force in horizontal direction

$(P_x) = 24 \times \pi/4 \times 5^2 \times 900$

$= 424.11$ KN

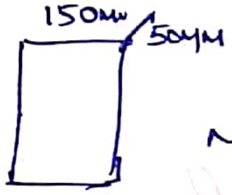
prestress force in vertical direction

$(P_y) = \pi/4 \times 5^2 \times 900$

$= 17.67$ KN

Horizontal stress $(f_x) = P_x/A_c = \frac{424.11 \times 10^3}{55 \times 10^3} = 7.7 \text{ N/mm}^2$

vertical stress $(f_y) = P_y/A_c = \frac{17.67 \times 10^3}{150 \times 50} = 2.35 \text{ N/mm}^2$



$V = 80 \text{KN} = 80 \times 10^3 \text{N}$

Max shear stress $(\tau_v) = \frac{VS}{Ib}$

$\Rightarrow \frac{80 \times 10^3 \times 468 \times 10^4}{189 \times 10^7 \times 50} = 3.96 \text{ N/mm}^2$

$\therefore f_{\substack{max \\ min}} = \left[\left(\frac{7.7+2.35}{2}\right) \pm \frac{1}{2}\sqrt{(7.7-2.35)^2 + 4(3.96^2)}\right.$

$= (5.025 \pm 4.77)$

$= 9.79 \text{ N/mm}^2, \; 0.255 \text{ N/mm}^2$

DESIGN OF SHEAR REINFORCEMENTS:-

As per (IS-1343-1980) the minimum shear r/f is provided in the form of stirrups such that

$S_v = \left(\frac{A_{sv} \cdot 0.87 f_y}{0.4b}\right)$ (pg: 48)

where, S_v – Spacing of stirrups

A_{sv} – Total c/s area of stirrups legs eff in shear

b – breadth of the member which for T, I & L-beams

f_y – Characteristic strength of stirrup (415 N/mm^2

If the shear force (V) is less than $0.5 V_c$ & in a member of minor importance, shear r/f need not be provided.

when V exceeds V_c, shear r/f is required conforming to the relation

$$S_v = \left[\frac{A_{sv}\, 0.87 f_y\, d_t}{V - V_c} \right] \quad [Pg:48]$$

$d_t \rightarrow$ depth from the extreme fibre to the longitudnal bars.

(uncracked in flexure) $V_c = V_{cw} = 0.67\, b_w h \sqrt{f_t^2 + 0.8 f_{cp} f_t}$ [pg:46]

P) The support section of a prestressed concrete beam 100mm wide & 250mm deep, is required an ultimate shear force of 60kN. The compressive strength of pre-stress at centroidal axis is $5 N/mm^2$. The characteristic strength of cube of concrete is $40 N/mm^2$. The cover to the tension r/f is 50mm. If the tensile strength of the steel in stirrups is $250 N/mm^2$, design suitable reinforcement at the section using the Indian standard code IS: 1343

Sol:- Given, $b_w = 100mm$ $\quad f_{ck} = 40 N/mm^2$

$h = 250mm$ $\quad f_y = 250 N/mm^2$

$d = ~~200~~ (250 - 50)$

$= 200mm$

$V = 60 kN$

$f_{cp} = 5 N/mm^2$

For the support section uncracked in flexure,

$V_c = V_{cw} = 0.67\, b_w h \sqrt{f_t^2 + 0.8 f_{cp} f_t}$

$\therefore f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40} = 1.52 N/mm^2$

$V_c = 0.67 \times 100 \times 250 \sqrt{1.52^2 + (0.8 \times 5 \times 1.52)}$

$= 48518 N$

$= 48.5 kN$

Use 6mm ϕ two legged stirrups.

$$S_v = \left[\frac{A_{sv}\, 0.87\, f_y\, d}{V - V_c} \right]$$

$$= \left[\frac{2 \times \pi/4 \times 6^2 \times 0.87 \times 250 \times 200}{(60 - 48.5) \times 10^3} \right]$$

$$= 213.3\,mm \approx 215\,mm$$

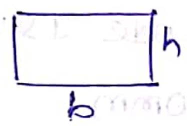
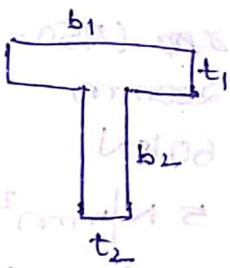
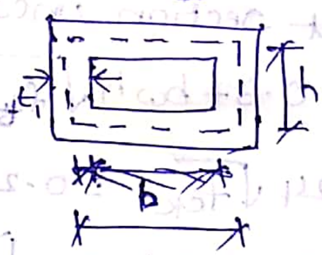
∴ Max permissible spacing $= 0.75\,d$

$= 0.75 \times 200$

$= 150\,mm$

∴ Adopt 6mm ϕ two legged stirrups @ 150mm c/c

Shear & principal stresses due to Torsion:-

S.NO	Name of the Section	Shape of the Section	Maximum shear stress
1.	Circle		$16T/\pi D^3$
2.	Rectangle		$(T/\alpha b^2 h)$ $\alpha \to 0.208 - 0.333$ as b/h varies from '1 to ∞'
3.	Flanged sections		$(3Tt_i / \Sigma b_i t_i^3)$
4.	BOX sections		$(T / 2At_i)$ where $(A = bh)$

8) The c/s of a prestressed concrete beam is r[illegible] with a width of 350mm and an overall depth of 700mm. The prestressing force of 180KN acts at eccentricity of 190mm. If the bending & twisting moments at the section are 80 & 20 KN m respectively, calculate the maximum principal tensile stress at the section.

In beams subjected to combined tension & bending, the middle of the bottom face is the critical point where the principal tension is maximum.

Sol:- Given, Area (A) $= 350\times 700$
$= 245\times 10^3\ mm^2$

$$I = \frac{350\times 700^3}{12} = 1.0\times 10^{10}\ mm^4$$

Shear stress due to torque at the soffit of the beam is given by.

$$\tau_t = \frac{T}{\alpha b^2 h}$$

$\frac{\cancel{750}}{\cancel{350}} =$

$$= \frac{20\times 10^6}{0.24\times 700^2\times 350} = 0.47\ N/mm^2$$

$\cancel{710}/300 = 2 \Rightarrow \alpha = 0.246$

from table of Seely & Smith

$\therefore$ Compressive stress due to prestressing force

$$f_b = \left(\frac{180\times 10^3}{245\times 10^3}\right) + \left[\frac{180\times 10^3\times 350\times 190}{10^{10}}\right]$$

$= 1.9\ N/mm^2$

$\therefore$ Tensile stress at the soffit due to bending moment

$$= \left(\frac{80\times 10^6\times 350}{10^{10}}\right) = 2.8\ N/mm^2$$

$\therefore$ Resultant direct stress at soffit $= 1.9 - 2.8$
$= -0.9\ N/mm^2$ (tension)

Principal tensile stress at the soffit

$$f_{min} = \left[\left(-0.9/2\right) - \frac{1}{2}\sqrt{-0.9^2 + 4(0.47^2)}\right]$$

$= [-0.45 - 0.13] = -0.58\ N/mm^2$ (tension)

6. TRANSFER OF PRESTRESS IN PRE TENSIONED MEMBERS

Factors:- The transmission of pre-stressing in members depends upon,

(i) Transmission of pre-stressing by bond
(ii) Transmission length
(iii) Bond stress
(iv) Transverse tensile stress
(v) End zone r/F
(vi) Flexural bond stress

Transmission by bond:-

The pre-stressing force in steel is transferred to concrete through the bond having

a) Adhesion
b) Friction
c) shearing resistance.

* The zone of transverse compression attains maximum bond stress.
* The bond-stress in the pre-stress mainly depends due to friction & shearing resistance as the bond stress at intermediate points is resisted by adhesion & in transfer zone the adhesion is destroyed by invariable slip & sink of tendons into concrete.
* when the bond stress is zero, the steel & concrete reach their maximum st. values of stresses with uniform stress distribution in this section.
* The length required to attain uniform stress distribution is known as "transmission length".

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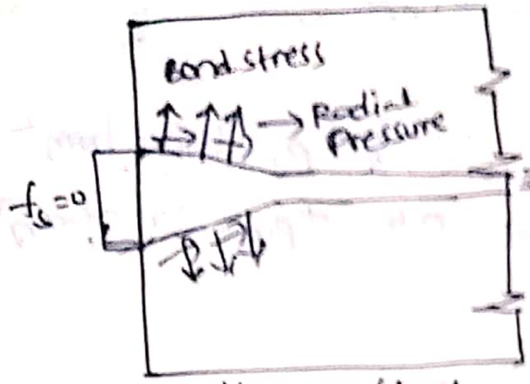
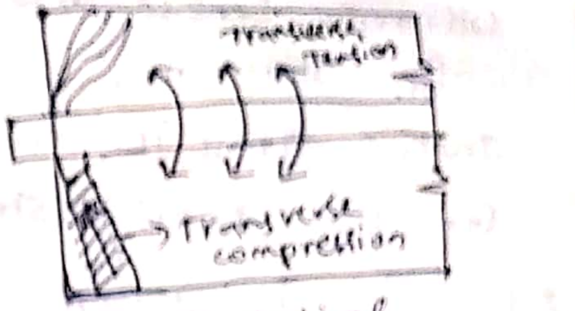
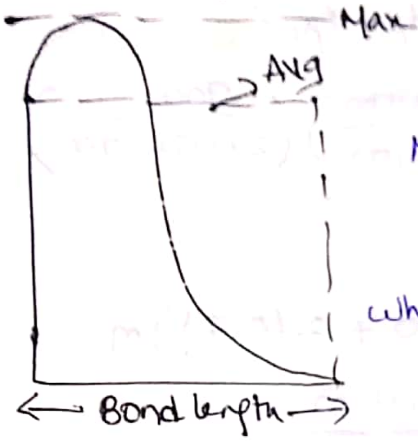


Hoyer effect



zone of radial Tranlverse compression



Based on wedge action, Hoyer has developed an expression

$$L_t = \frac{\phi}{2\mu}(1+\nu_c)\left(\frac{\alpha_e}{\nu_s} - \frac{f_{pi}}{E_c}\right)\left(\frac{f_{pe}}{2f_{pi}-f_{pe}}\right)$$

where, L_t = transmission length
ϕ = wire diameter
μ = coefficient of friction b/n steel & concrete
ν_c = poission ratio for concrete
ν_s = " " " steel
α_e = modular ratio (E_s/E_c)

E_c = modulus of Elasticity
f_{pi} = initial stress in steel
f_{pe} = eff stress in steel

* The empirical formulae for transmission length

$$\left\{ L_t = \sqrt{\frac{\sqrt{f_{cu}} \times 10^3}{\beta}} \right\}$$

where, f_{cu} → cube strength of concrete at transfer
β = constant, depends upon strads & wires

P) Calculate the transmission length at the end of a pre-tensioned beam as per Hoyer's method using the following data

Span of the beam = 50m
Dia of wires used = 7mm
coefficient of friction b/n steel & concrete = 0.1
Poisson's ratio for steel = 0.30
poisson's ratio for concrete = 0.15
E_s = 210 kN/mm² & E_c = 30 kN/mm².

Ultimate tensile strength of steel wire,

$f_{pu} = 1500\ N/mm^2$

Initial stress in steel, $f_{pi} = 0.7 f_{pu} = 1050\ N/mm^2$

effective stress in steel, $f_{pe} = 0.6 f_{pu} = 900\ N/mm^2$

Sol: using Hoyer's eq'n

$$L_t = \frac{\phi}{2\mu}(1+\nu_c)\left(\frac{\alpha_e}{\nu_s} - \frac{f_{pi}}{E_c}\right)\left(\frac{f_{pe}}{2f_{pi} - f_{pe}}\right)$$

$$= \frac{7}{2\times0.1}(1+0.15)\left(\frac{7}{0.3} - \frac{0.7\times1500}{30\times10^3}\right)\left(\frac{900}{2\times1050 - 900}\right)$$

$$= 700\ mm$$

∴ Total length of beam $= (50 + 2(0.7))\ m$

$= 51.4\ m$

P) Estimate the transmission length at the ends of a pre-tensioned beam prestressed by 7mm ϕ wires, Assume the cube strength of concrete at transfer as $42\ N/mm^2$

Sol:

$$L_t = \sqrt{\frac{\sqrt{f_{cu}} \times 10^3}{\beta}}$$

for 7mm ϕ wires $\beta = 0.0174$ & $f_{cu} = 42\ N/mm^2$

$$L_t = \sqrt{\frac{\sqrt{42}\times10^3}{0.0174}} = 610\ mm$$

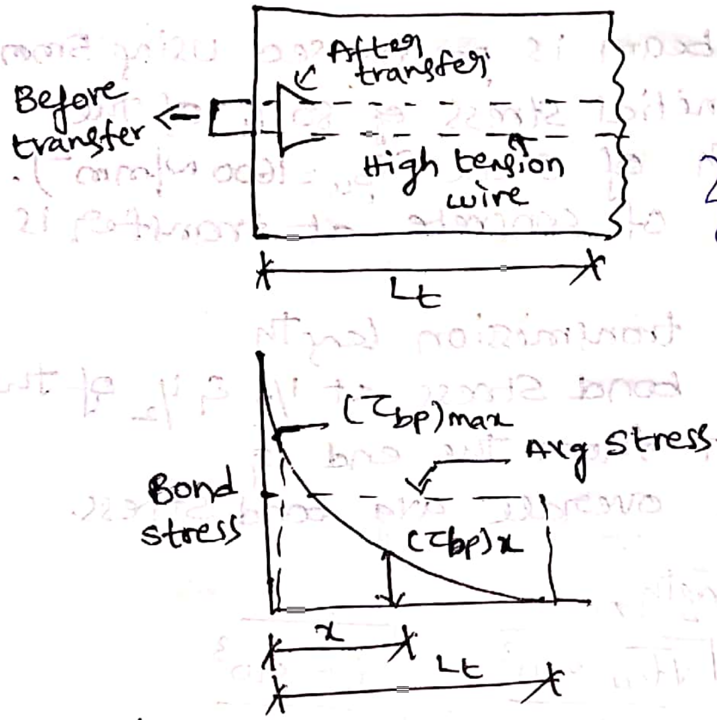
If on other hand, 5mm ϕ wires ~~are~~ are used,

$\beta = 0.0235$

$$L_t = \sqrt{\frac{\sqrt{42}\times10^3}{0.0235}} = 525\ mm$$

Bond stresses:-

The magnitude of bond stresses developed b/n concrete & steel and its variation in the transfer zone of pre-tensioned beams as shown in figure. The bond stress is zero at the ends but builds up rapidly to a maximum over a very short length. This value decreases as the stress in the wire builds up. At a distance equal to transmission length, the bond stress is almost zero while the stress in steel & concrete reach their maximum values.



$$\left\{(\tau_{bp})_{Avg} = \left[\frac{19.6 \times \sigma_i \times f_{pu}}{\pi \times \phi \times L_t}\right]\right\}$$

Fig:- Based on stress in pretensioned Beams

If $(\tau_{bp})_{max}$ = max. value of bond stress

$(\tau_{bp})_x$ = bond stress at a distance 'x' from the free end

ϕ = dia of the wire

f_s = stress in steel at a distance 'x' from the free end

f_{se} = effective stress in steel at the end of transfer zone

Based on test, the relation have been proposed as

$$\left\{(\tau_{bp})_x = (\tau_{bp})_{max}\, e^{-4\psi x/\phi}\right\}$$

$$\left\{f_s = f_{se}\left(1 - e^{-4\psi x/\phi}\right)\right\}$$

where, ψ = constant, expressed as the ratio of change in bond stress to steel stress

x = distance measured from the free end, in (mm)

Based on test using wires of 2 & 5mm ϕ stressed to 1575 & 1100 N/mm^2 respectively having a cube crushing strength of 80 N/mm^2, the values of max. bond stress $(\tau_{bp})_{max}$ & constant were found to be 7.42 N/mm^2 and 0.00725 resp.

P) A pre-tensioned beam is prestressed using 5mm ϕ wires with an initial stress of 80% of the ultimate strength of steel ($f_{pu} = 1600\ N/mm^2$). The cube strength of concrete at transfer is 30 N/mm^2.

a) calculate the transmission length
b) compute the bond stress at 1/4 & 1/2 of the transmission length from the end &
c) calculate the overall avg bond stress.

Sol: Transmission length,

$$L_t = \sqrt{\frac{\sqrt{f_{cu}} \times 10^3}{\beta}} = \sqrt{\frac{\sqrt{30} \times 10^3}{0.0235}}$$

$$= 482.77 \approx 485mm$$

Bond stress is given by,

$$(\tau_{bp})_x = (\tau_{bp})_{max} \cdot e^{-4\beta x/\phi}$$

$$= 7.42 \cdot e^{-(4 \times 0.00725 \times x)/\phi}$$

If $\phi = 5mm$, then

$$(\tau_{bp})_x = 7.42 \cdot e^{-0.0058x}$$

Bond stress at 1/4 is given by

$$L_t/4 = 485/4 = 121.25mm$$

$$\therefore \tau_{bp} = 7.42 \cdot e^{-0.0058 \times 121.25}$$

$= 3.67\ N/mm^2$ (at 121.25 mm from the end)

Bond stress at 1/2 is given by

$$L_t/2 = 485/2 = 242.5mm$$

$$\tau_{bp} = 7.42 \cdot e^{-0.0058 \times 242.5} = 1.81\ N/mm^2$$

(at 242.5 mm from end)

overall Avg bond stress is given by

$$\tau_{bp}(Avg) = \left[\frac{19.6 \times 0.8 \times 1600}{5 \times \pi \times 485}\right] = \underline{3.3\ N/mm^2}$$

Transverse Tensile stresses:-

* The transverse tensile stresses in transmission zone is developed due to concentration of tendons at the ends.
* It was induced methods of jacking etc.
* The area (or) near centroidal section of the end faces of beam generally have maximum tensile strength of concrete.
* Horizontal cracking occurs if the tensile stress exceeds the tensile strength of the concrete
* It was determined by

$$\left\{ f_v = \frac{kM}{b_w d^2} \right\}$$

where, $f_v \rightarrow$ Transverse tensile stress at centroid of the end face

$M \rightarrow$ Resulting B.M b/n prestress force & internal pre-stress

$b_w \rightarrow$ web thickness

$d \rightarrow$ overall depth of beam

$k \rightarrow$ constant depending upon the slope & distribution of tendons at the ends

→ The transverse tensile stress distribution in transfer/ transmission zone is given by

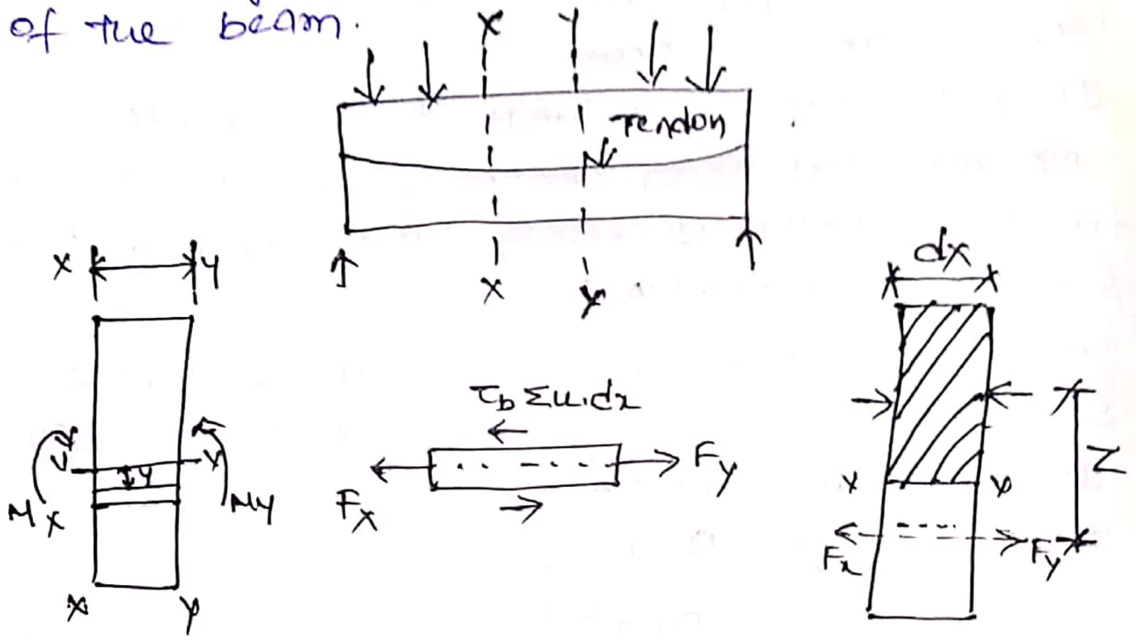
$$f_v = \frac{10M}{b_w h L_t}\left(1 - \frac{x}{L_t}\right)e^{-3.5 \times L_t}$$

where, L_t = Transmission length

x = Distance of end face.

Flexural Bond stresses:-

The bonded tendons in pre-tensioned & post-tensioned beams generate the bond stress b/n steel & concrete when the sections are subjected to the transverse shear forces due to the rate of change of moment along the length of the beam.



a) uncracked members

Fig:- Flexural Bond stresses

b) cracked members

where, from fig

τ_b = Bond stress b/n steel & concrete

V = Shear force

M_x, M_y = moments at the section XX & YY

Σu = Total perimeter of tendons

Y = Distance of tendons from the centroidal axis

I = Second moment of area of section

α_c = modular ratio = E_s/E_c

A_s = Area of steel

f_x & f_y = Bending stress in concrete at the level of steel at the section XX & YY considering the forces & moments from the fig. 8(A)

$$M_x - M_y = V \cdot dx = \left[\left(f_y \cdot \frac{I}{y}\right) \times \left(f_x \cdot \frac{I}{y}\right)\right]$$

$$V \cdot dx = \left(\frac{I}{\alpha_e A_s Y}\right)(\alpha_e A_s f_y - \alpha_e A_s f_x)$$

$$V \cdot dx = \left(\frac{I}{\alpha_e A_s Y}(F_y - F_x)\right)$$

$$V\, dx = \left(\frac{I}{\alpha_e A_s Y}\right)[\tau_b \cdot \Sigma u\, dx]$$

$$\left\{ \therefore \tau_b = \left(\frac{\alpha_e A_s Y V}{I \Sigma u}\right) \right\}$$

If round wires are used,

$$\frac{A_s}{\Sigma u} = \frac{\phi}{4}.$$

where, ϕ = dia of the wire

Then, $$\left\{ \tau_b = \left(\frac{\alpha_e \cdot V \cdot Y \phi}{4I}\right) \right\}$$

In case of cracked flexural members, bond stresses change suddenly at the cracks due to the abrupt transfer of tension from concrete to steel. The bond stresses gradually reduces to a minimum value in b/n the cracks.

considering cracked sections of a beam of length 'dx' as shown in fig

$$\{ V\, dx = (F_y - F_x) \cdot Z \}$$

If τ_b = bond stresses developed

$$V \cdot dx = (\tau_b \Sigma u \cdot dx) Z$$

$$\therefore \left\{ \cancel{\tau_b = \frac{V}{\Sigma u \cdot Z}} \right\} \quad \left\{ \tau_b = \frac{V}{Z \cdot \Sigma u} \right\}$$

P) A post-tensioned prestressed concrete rectangular beam, 240mm wide by 500 mm depth, is grouted before the application of liveloads. The steel consists of three tendons, each made up of 12 ~~members~~ numbers of 7mm ϕ wire encased in a thin metallic hose of 30mm dia wire with an effective cover of 50 mm. The modulus of elasticity of steel & concrete are 210 & 35 kN/mm² respectively. The beam spans 10m & supports two concentrated loads of 250 kN each at the third points. Compute the unit bond stress.

a) b/n each wire & grout; &

b) b/n ~~each~~ the hose & the concrete.

Sol: Max. Shear force in the beam, $V = 250kN$

Second moment of area, $I = \frac{bd^3}{12}$

$= \frac{240 \times 500^3}{12}$

$= 25 \times 10^8 \, mm^4$

modular ratio, $\alpha_e = E_s/E_c$

$= 6$

dist of tendons from centroidal axis $\} = y = 200mm \quad \left(\frac{500}{2} - 50\right) = 200mm$

a) Bond b/n each wire & grout $= \frac{V \cdot y \alpha_e \phi}{4I}$

$= \frac{250 \times 10^3 \times 200 \times 6 \times 7}{4 \times 25 \times 10^8} = 0.21 \, N/mm^2$

b) Area of steel in on hose,

$A_s = 12 \times 38.5 = 462 mm^2$

Hose diameter $= 30mm$

Hose circumference $= \pi \times 30 = 94.24 \, mm$

Bond stress b/n hose & concrete $= \frac{V \alpha_e A_s y}{\Sigma u \cdot I}$

$= \left(\frac{250 \times 10^3 \times 6 \times 462 \times 200}{94.24 \times 25 \times 10^8}\right) = \underline{0.588 \, N/mm^2}$

P) A pre-tensioned beam of $\square^{lar}$ section, with a width of 200 mm & 500mm overall depth, is prestressed by 5 wires of 7mm dia located 100 mm from the soffit. The maximum shear force at a particular section is 100 kN. If the modular ratio is 6, calculate the bond stress developed, assuming a) The section ~~as~~ is uncracked, &
b) " " is cracked.

Sol: a) If section is uncracked,

Bond stresses, $\tau_b = \left(\frac{V \cdot y \, \alpha_e \, \phi}{4I} \right)$

$V = 100 \text{ kN} = 100 \times 10^3 \text{ N}$

$y = \cancel{150} \; \frac{500}{2} - 100 = 150 \text{ mm}$

$\alpha_e = 6 \; ; \quad \phi = 7\text{mm}$

$I = \frac{bd^3}{12} = \cancel{200} \; \frac{200 \times 500^3}{12} = 2.08 \times 10^9 \text{ mm}^4$

$\therefore \tau_b = \frac{\alpha_e \cdot y \cdot V \cdot \phi}{4I}$

$= \cancel{6 \times 150 \times} \; \frac{6 \times 150 \times 100 \times 10^3 \times 7}{4 \times 2.08 \times 10^9}$

$= \underline{0.075 \text{ N/mm}^2}$

b) If section is cracked,

Bond stress, $\tau_b = \frac{V}{\Sigma u \cdot z}$

where, $z \rightarrow$ lever arm, assume 7/8 times of eff. depth

$\therefore \tau_b = \frac{100 \times 10^3}{\left(\frac{7}{8} \times 400\right) \times 5 \times \pi \times 7}$

$= 2.59 \text{ N/mm}^2$

$\approx 2.6 \text{ N/mm}^2$

$\therefore \left\{ \tau_b = 2.6 \text{ N/mm}^2 \right\}$